Speedups for Efficient Genetic Algorithms : Design optimization of low-boom supersonic jet using parallel GA and micro-GA with external memory

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ABSTRACT

GAs have been successfully used in an aerodynamic shape design optimization. Time efficiency issues related to the evaluation of a fitness are becoming a critical point as large calculations are needed. In this paper two efficient methods are applied to the original GAs to save computational time. Firstly, parallelized GA is used for a single disciplinary optimization to investigate the scalability corresponding to the number of processors. Without other disciplines, only a minimization of sonic boom is sought using a parallelized GA.

The minimization of the sonic boom ground signature often leads to the undesirable aerodynamic properties such as increase of the drag. These disciplines are often conflicting, and it is important to balance aerodynamic performance and sonic boom requirements in a way that represents the best compromise for the overall design.

Therefore, secondly, multi-objective methodology can be used to find a set of non-dominated solutions called the optimal Pareto sets for minimizing drag and sonic boom at the same time. Non-dominated sorting GA is used to get these optimal Pareto sets. To decrease the increase of computation time caused by adding more disciplines and comparing the ranks each other, micro-GA with external memory is inserted in the original non-dominated sorting GA.

Results of a shape optimization of a low-boom supersonic jet with 15 design variables are presented. CPU time in running a parallel GA and micro-GA with external memory is compared with a serial GA and a usual non-dominated sorting GA respectively. Both methods are shown to be successful by decreasing run time significantly.

1. INTRODUCTION and BACKGROUND

For decades, the development of economically and environmentally acceptable supersonic aircraft has been identified as a key step toward the next generation of aviation history that could improve many aspects of human life and foster economic growth. The key technology barrier for this class of aircraft is the elimination, or reduction to acceptable levels, of the sonic boom for flight over land while guaranteeing the challenging performance requirements of the other major disciplines. This essentially means that all major disciplines including aerodynamics, structures, stability and control, mission, and propulsion should be taken into account from the early stages of design process; therefore, boom reduction must be considered as an additional aspect of the multi-discipline optimization (MDO) problem.

Since the ground sonic boom is typically not a smooth function of the design variables and may actually contain multiple local minima, it is important to select an optimization algorithm that is able to cope with this kind of design space. Since GAs do not require sensitivity information, they are ideally suited to this kind of realistic design environments where discontinuities, multimodality and noisy response may exist. They also have considerable advantages for multi-objective design problems to obtain Pareto optimal sets. But major drawback of the GA approach is that it requires many generations to locate the global optimum point and consequentially is quite computationally demanding with large number of evaluations. In addition, accurate prediction of aerodynamic performance like C_L (lift coefficient) and C_D (drag coefficient) as well as sonic boom ground boom signature usually requires very expensive three dimensional Euler Flow Solver in computational fluid dynamics (CFD).

Therefore, significant improvement in several aspects should be implemented to make use of the GAs accurately and efficiently. For the improvement of GA itself, parallelized GA can reduce the computation time to evaluate the individual objective functions, by distributing jobs to many processors. According to Gordon and Whitley [6], parallel GAs can be classified according to two different models in terms of accessing the global population. Island model uses multiple subpopulations and cellular model (often called a fine-grained model) employs partitioning of a single population. Section 4.1 gives a brief overview of these two forms of parallelism used in parallel GAs and of the island model in particular. In this study, I used PGApack parallel genetic algorithm library made by Argonne National Library[10]. PGApack is applied to minimize the sonic boom ground signature of supersonic jet and this problem can be seen single discipline optimization problem.

But real design problem usually consists of rather multi-disciplines with conflicting objective functions than single discipline. To preserve or enhance the aerodynamic properties such as C_L and C_D, while minimizing sonic boom, GA appropriate to multi-criteria optimization is necessary.

In a multi-objective optimization problem, the notion of optimality is not so obvious. If we agree in advance that we cannot link the values of the different objectives, we must find a different definition of optimality, a definition which respects the integrity of each criterion. Then, the concept of Pareto optimality arises.

There is no solution that is the best for all the criteria, but there exists a set of solutions that are better than other solutions in all the search space, when considering all the objectives. This set of solution is known as the optimal solutions of the *Pareto set* or *non-dominated solutions* [1]. The usual approach has been to use a ranking procedure to classify a population of individuals based on their Pareto dominance. One of this popular methods is Non-dominated Sorting Genetic Algorithm (NSGA) based on the algorithm of Srinivas and Deb[16]. This ranking procedure normally consumes most of the running time of an evolutionary multi-objective optimization technique [2]. Several researches have focused their recent efforts on reducing the checking for nondominance and in the development of efficient approaches to keep diversity. Micro-GA with the use of external files is suggested by Carlos [3]. Details are explained in section 4.2.

In this paper, I used both of NSGAII and Micro-GA with external memory to compare the performance of those multi-objective optimizers both in terms of time required to find a solution and the quality of the solution obtained.

Finally, for improvement of function evaluation tools, expensive CFD-based computation method should be replaced with the accurate approximation models. In this study, Kriging approximation model was used.
Since this CFD issue isn't the main purpose of this paper, only short descriptions are given in section 3.

2. STATEMENT OF PROBLEM

Configuration of interest : **Low-Boom Supersonic Business Jet(SBJ) Design**

The design problem in question involves the simultaneous ground boom and drag minimization of a supersonic business jet wing-body-tail-nacelle for the configuration at a specified lift coefficient, $C_L = 0.07104$, which corresponds to a cruise weight of 100,000 lbs at a cruise altitude of 50,000 ft. The wing reference area is 1,032 ft^2. The free-stream flow conditions were fixed at Mach number = 2.0 which means twice of the sound speed. The aircraft geometry and flow conditions were parameterized directly in CAD using 108 design variables [4]. The list of geometric design variables and their upper and lower bounds for the 15-dimensional design problem which we will describe later is given below:

x_1 = wing position along fuselage	$\in$ [49.643, 65.278]ft
x_2 = wing dihedral angle	$\in$ [-0.273, 3.611] deg
x_3 = wing sweep angle	$\in$ [26.856, 41.610] deg
x_4 = wing aspect ratio	$\in$ [4.643, 7.148]
x_5 = wing leading edge extension	$\in$ [0.776, 1.244]
x_6 = upper fuselage radius at 12.50\% of fuselage length	$\in$ [2.33, 3.0321]
x_7 = upper fuselage radius at 18.75\% of fuselage length	$\in$ [3.1633, 3.9967]
x_8 = upper fuselage radius at 25.00\% of fuselage length	$\in$ [3.4905, 4.6384]
x_9 = upper fuselage radius at 31.25\% of fuselage length	$\in$ [3.3081, 4.6058]
x_{10} = upper fuselage radius at 37.50\% of fuselage length	$\in$ [3.3015, 4.6177]
x_{11} = lower fuselage radius at 12.50\% of fuselage length	$\in$ [2.33, 3.0321]
x_{12} = lower fuselage radius at 18.75\% of fuselage length	$\in$ [3.1633, 3.9967]
x_{13} = lower fuselage radius at 25.00\% of fuselage length	$\in$ [3.4905, 4.6384]
x_{14} = lower fuselage radius at 31.25\% of fuselage length	$\in$ [3.3081, 4.6058]

x_{15} = lower fuselage radius at 37.50\% of fuselage length $\in$ [3.3015, 4.6177]

The wing planform for this configuration was designed with experience from our previous supersonic work to have a portion with a subsonic leading edge followed by an outboard panel with a supersonic leading edge. This helps increase the span to achieve better low-speed performance at a small cost in cruise performance. The airfoil sections for the outboard portion of the main wing and the horizontal tail were chosen to be simple biconvex airfoils of varying thickness, while an RAE2822 was used for the inboard part of the main wing with a subsonic leading edge. Surface mesh triangulation is shown at fig.1 to simply demonstrate baseline configuration.

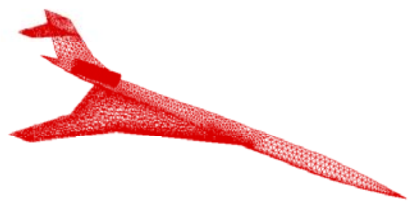


Fig.1 Surface triangulation of baseline configuration

Specifications of problems in GA

All the fifteen design variables have their own bounds for the realistic aerodynamic shape and they are obtained by experiments performed before. Fig.1 shows the baseline configuration corresponding to initial setup for 15 design variables, i.e. {55, 1, 35, 6, 1, 2.68, 3.6, 4.0, 4.0, 4.0, 2.68, 3.6, 4.0, 4.0, 4.0}. Lower and upper bounds for all the design variables are normalized between 0 and 1 to be compatible with a randomizing function in the GA. All the alleles are set to the values selected uniformly and randomly from the specified intervals. Total count depends on how uniformly and randomly the alleles are picked in the interval. If I assume to have only three choices for each interval, then total count would be $3^{15.}$

Objective functions are the drag coefficient and sonic boom ground signature calculated either from exact CFD analysis or alternative Kriging approximation model. The Drag coefficient and sonic boom ground signature for the baseline configuration is about 0.01 and 0.6 psf or so. I can assume all the possible objective function values with realistically constrained design variables doesn't deviate much from those values. The goal is to achieve minimum sonic boom and drag coefficient. Since we don't know the exact minimum values in advance, my termination criteria is set as the number of no changes in the best solution found through several generations, in single discipline case. But in the multi-discipline case, the termination criteria is dependent of decision maker's option with the trade-offs between the objectives. In this study, I set the maximum generation number as the stopping criteria and investigated the Pareto sets afterwards.

Both of the GAs use common genetic operators like reproduction, crossover and mutation. For the multi-objective optimization case, there were more stages than those of the original GAs such as ranking and elitism. The details of those stages are explained in section 4.2.

3. FUNCTION EVALUATION

High-fidelity three dimensional Euler Flow Solver

One of high-fidelity CFD analysis tool, an Euler Flow Solver, is developed in my laboratory by incorporating CAD-based geometry engine, unstructured, adaptive meshing techniques [5, 7]. This analysis is fully parallelized with a partitioned computation domain and all the independent modules are automated. But as stated in the introduction, the accurate function evaluation using this tool is very expensive. This high-fidelity computation tool needs at least several minutes of computation to evaluate the accurate C_L, C_D and sonic boom ground signature corresponding to each individual. Therefore it is almost impossible to connect directly Euler Flow Solver with GAs. Faced with these problems, the alternative to the high-fidelity CFD analysis has received increased attention in recent years. A response surface method, Kriging approximation method and Cokriging approximation are widely used. Detailed description of these techiniques are not the main concerns of this study and they are given in [7]

Kriging approximation model alternative to expensive CFD analysis

One of the alternative to the expensive CFD analysis is the Kriging approximation model [8] which is developed in statistics and geostatistics, in order to approximate the results of deterministic computer analysis. This is built by interpolation of the sampled data by maximum likelihood estimation procedure, which allows for the capturing of multiple local extrema. The cost of Kriging method is orders of magnitude smaller than the expensive CFD. But the accuracy of a Kriging model depends greatly on the number of sample data points used and their location in multidimensional space.

In this study, using the latin hypercube sampling technique, 140 sampling points around the baseline design points in the design space were selected and their performance values were computed using CFD analysis. A Kriging model was then generated based on the sampled data and used for the function evaluation routine within the several GAs used in this study. Validation of results with CFD calculation is another issue but I didn't include it in this study. All the numerical specifications and details are in [7].

4. METHODS

4.1 Parallel GA for single disciplinary optimization

Several approaches grew out of the desire to easily parallelize GAs, one is the cellular model. The cellular model [13] uses fine grained, massively parallel architectures. These machines consist of a huge number of simple processors typically connected in a ring or torus topology. One individual is assigned to each processor. Selection and crossover is restricted to local neighborhoods of a particular processor. This model requires specialized massively parallel computers and a large amount of communication.

The second approach is the parallel island model. The island model is designed to use a coarse grained parallel architecture. Each processor is given a population of individuals. The processors evolve their populations using a serial GA. Each GA is identical to but independent of the others. Each GA is usually started with a different random seed. Periodically a processor may migrate a number of its individuals to another population. The amount of communication involved in the island model appears to be much more manageable than the cellular models. This island model seems much appropriate in my study considering limited numbers of processes.

In this work, PGAPack written by Argone National Laboratory is used for parallel island model. One of the limitations for the PGAPack is that it doesn't use migration stage through which individuals can be transmitted between the islands to communicate each other when necessary.

PGAPack is general purpose, data-structure-neutral, parallel genetic algorithm library. Its key features on which we are interested are:

1. Runs on uniprocessor, parallel computers , and workstation networks.
2. Binary-, integer-,real-, and character-valued native data types.
3. Full extensibility to support custom operators and new data types.
4. Parameterized population replacement.
5. Easy-to-use interface for novice and application users.
6. Multiple choices for selection, crossover, and mutation operators.
7. Large set of example problems
8. Easy integration of hill-climbing heuristics.

Tableau for sonic boom minimization is below.

Objective	Minimize the sonic boom ground signature
Representation scheme	• Structure = combination of 15 design variables • K = • L = 15 • Mapping from points in search space of the

	problem to structures in the population = each allele is normalized floating point presentation between 0.0 and 1.0
Fitness cases	Same as fitness
Fitness	Sonic boom ground signature (dp/p =(p-p0) / p)
Parameters	Population size M = 1000, G=2000
Termination criteria	If no change of best individuals are larger than 40
Result designation	

Table. 1 Tableau for single discipline of sonic boom minimization

4.2 Micro-GA with external memory for multi-disciplinary optimization

Many real-world optimization problems, especially in MDO situations, require the simultaneous optimization of possibly conflicting multiple objectives: this approach is often referred to as *multiobjective optimization*. Unlike single-objective optimization where only one optimal solution is pursued, a typical multiobjective optimization problem produces a set of solutions which are superior to the rest with respect to all objective criteria, but are inferior to other solutions in one or more objectives. These solutions are known as Pareto optimal solutions or non-dominated solutions. None of the solutions in the Pareto optimal set is absolutely better than any others with respect to all of the objectives being considered; therefore, any one of them is an acceptable solution. Once the set of optimal solutions is identified, it is left to the designer to choose one solution out of the many possible ones.

A genetic algorithm can use this dominance criteria in a straightforward fashion to drive the search process toward the Pareto front. Due to the unique features of GAs, which work with a population of solutions, multiple Pareto optimal solutions can be captured in a single run. This is the primary reason that makes GAs ideally suited for multi-objective optimization.

But this ranking procedure normally consumes most of the running time of an evolutionary multi-objective optimization technique [2]. Pareto ranking is O (kM^2), where k is the number of objective functions and M is the size of the population. Additionally, an extra mechanism is required to preserve diversity. This generally implies the use of another process that is O (M^2).

A recent study by Coello [14] proposed a micro-GA-based multi-objective optimization that uses an external file of non-dominated vectors found in previous generations to accelerate the multi-objective optimization process. The method implemented an additional elitism strategy and an adaptive grid-type technique[17] to accelerate the convergence and to keep the diversity in the Pareto front. The Micro-GA algorithm is a specialized GA that works with a very small population size of usually 3-6 and a reinitialization step. It has a well-known property of faster convergence rates than other GAs. In the present research, some of the ideas of Coello's work have been adapted to a single objective micro-GA algorithm along with the traditional Goldberg's Pareto ranking approach in order to develop an efficient and robust design framework. The authors have modified a micro-GA algorithm originally developed by Carroll [15].

The procedure is illustrated in Figure 2. First, a random population is generated and their objective values are calculated as in the original micro-GA. Then, to ensure that all the non-dominated individuals have same level of reproductive potential, Goldberg's non-dominated sorting procedure is implemented. Therefore, the fitness level of each individual is determined based on the non-domination criterion rather than the objective function value itself.

Based on the rank of non-dominance, the population goes through the usual operations of micro-GA, namely selection and crossover, and check to see if the nominal convergence among the population points has been reached. If the problem is not converged, the algorithm returns to the function evaluation and non-dominance ranking steps for the new generation, otherwise it continues on to the reinitialization step.

Two types of elitism are implemented in the reinitialization step. The first type carries on the best solutions from the previous nominal convergence stage. This is the same elitism strategy used in the single objective micro-GA. The second type involves the storing of non-dominated vectors produced from each cycle of micro-GA to an external file and inserting some of the best solutions generated so far in the reinitialized population for the micro-GA. This process is applied at regular intervals to improve the non-dominated solutions by getting closer to the true Pareto front or by achieving a better distribution.

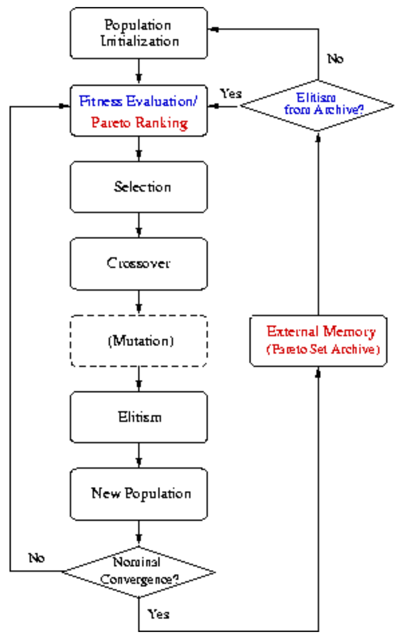


Fig. 2 Flow Chart for Multiobjective GA

Representation scheme	• Structure = combination of 15 design variables • K = • L = 15 • Mapping from points in search space of the problem to structures in the population = each allele is normalized floating point presentation between 0.0 and 1.0
Fitness cases	Vector of two objective functions in fitness
Fitness	Sonic boom ground signature (dp/p =(p-p0) / p) and Drag coefficient
Parameters	Population size M = 200, G=400
Termination criteria	If it reaches maximum generation number.
Result designation	

Table. 2 Tableau for multi disciplines of minimization of sonic boom and drag coefficient

5. **RESULTS and DISCUSSION**

Single-discipline optimization

I used PGApack library to implement a typical serial GA and parallel GA using the island model. Since the same library is used in both GAs, the GA used on each processor in the island model was identical to the one used in the serial GA. The parallel GA was implemented at a Silicon Graphics workstation which has 32 processors. Since Kriging-based approximate function evaluation was computationally efficient, running time was not much even for the serial GA. Because of the little computational burden of the problem, the expected theoretical linear scalability wasn't exactly achieved in this problem. But still CPU time decreases as the number of processors increases.

Table. 1 shows the tendency that as population size increases, computation time increases as well. But the best fitness values are not much different one another. So population size 500~1000 can be accepted as an optimal size for this problem. I tried the calculations with population size 2000, but a small amount of improvement of fitness values was achieved.

Fig. 3 shows the fitness history for the best and average. Due to the slow convergence rate at a later stage, faster convergent modification is desirable. Gradient information can be effectively used on that purpose. Hybrid scheme with gradient information-based local optimizer will be on the future work.

Population	Number of processors	CPU time (sec)	Iteration number	Best fitness
250	1	15.28	2780	4.553273 x 0.1
	2	9.3		
	4	6.72		
500	1	27.532	2530	4.551973 x 0.1
	2	16.775		
	4	11.923		
750	1	56.00	3450	4.551271 x 0.1
	2	32.778		
	4	23.0082		
	8	15.61		
1000	1	54.37	2500	4.551376 x 0.1
	2	32.87		
	4	23.07		
	8	16.303		

Table 1. Tableau for comparison of CUP time corresponding to number of processors at several population size

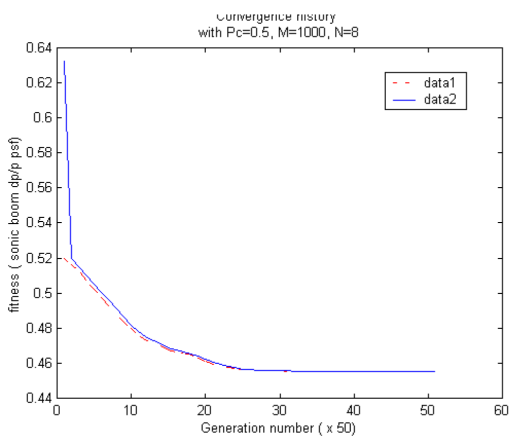


Fig. 3 Convergence history of best and average fitness

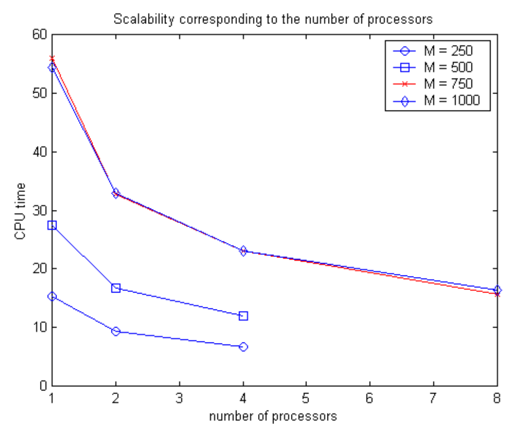


Fig. 4 CPU time corresponding to several processors at different population size

Best string with normalized design variables was {0.98235, 0.2397184, 0.6838408, 0.5419679, 0.993096, 0.3755819, 0.3325736, 0.1301846, 0.965388, 0.04221448, 0.335045, 0.9904073, 0.03419943, 0.4543103, 0.0004687757} and can be converted realistic values as {65.002, 0.658052, 36.9454, 6.00063, 1.24074, 2.5937, 3.44047, 3.63993, 4.56088, 3.35706, 2.56524, 3.98871, 3.52976, 3.89766, 3.30212}.

Multi-discipline optimization

Using a latin hypercube sampling technique, 140 sample points were selected, and their C_D and boom overpressure (normalized difference of pressure from reference pressure) were computed using CFD analysis. A Kriging model was then generated based on the sampled data and used for the function evaluation subroutines within the two types of multi-object GA(MOGA) search, Micro-GA with the use of external file usage and NSGAII. The estimated Pareto set from the Kriging-based MOGA search procedure is plotted as black in Fig. 5 and Pareto sets from NSGAII is shown at Fig. 6. Both Pareto sets have similar distribution of optimal points. If I put more points in the Pareto sets from micro-GA based GA, the fronts will be very similar each other.

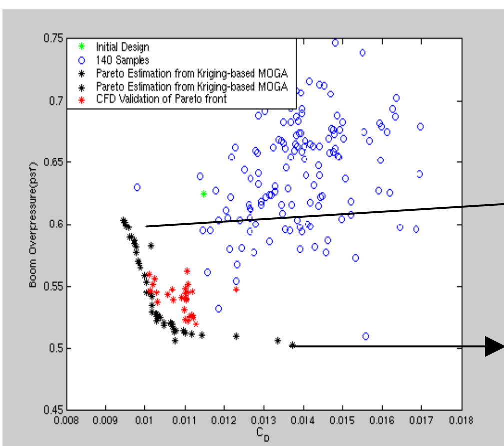
In Fig. 5, the 140 samples of initial function evaluations done by CFD analysis are dotted as blue for comparison purpose. CFD validation in red shows good agreement with the results from Kriging based approximation. If I investigate the optimized best ground boom signature, the results demonstrate the fact that the boom design space may have discontinuous or non-smooth regions and cause difficulties in generating accurate Kriging models. However, the estimation produced some good design points in terms of both design criteria.

The resulting design configuration with optimized design variables are plotted along with the points on the Pareto front. The wing sweep angle, leading edge extension, and dihedral angle increased while the wing position along the fuselage and wing aspect ratio decreased. The nose section was deformed such that it decreased the initial shock strength from the nose and generated an expansion region by inducing a bump-like shape at the lower fuselage section. The effect of this expansion wave is to both weaken the initial shock and to prevent the first and second shocks from coalescing.

CPU time corresponding to several types of the GAs for multi-objectives is evaluated and shown at table. 4. Even if a small amount of increment of population size caused significant increase of computation time, I set a small population size which I got through several experiments. With a bigger population size, the objective function values in Pareto set didn't change much. It is due to the increase of the computaion time consumed in ranking each individuals by nondominancy criteria. After several experiments with different population size, the minimum population size which has desirable optimal Pareto sets was set as 200. As we can expect, conventional non-dominated sorting GA such as number 1 and number 3 in table.4 has long CPU time, while Micro-GA with the use of external file has minimum CPU time in this comparison.

	1. NSGAII	2. Micro-GA with external use	3. Number 2 GA without Micro-GA option
CPU time (sec) Pop = 200 Maximum gen = 400	65.94	25.28	59

Table. 4 Comparison of CPU time corresponding several GAs for multi-objective functions.



Best Cd configuration

Best boom configuration

Fig. 5 Results of multiobjective GA(MOGA) Based on Kriging approximation model

Best ground boom signature

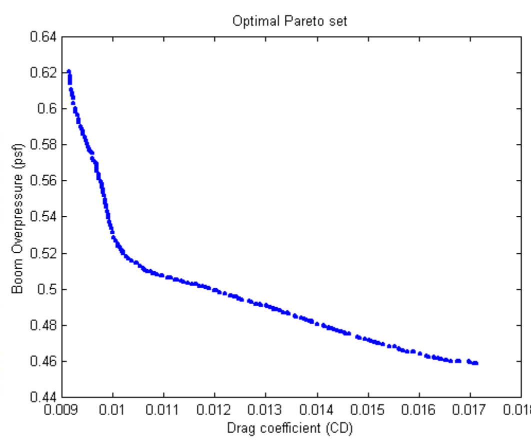


Fig. 6 Results from NSGAII

6. CONCLUSIONS and FUTURE WORKS

In this work, two ways of improving efficiency of conventional GAs are investigated in a problem of design optimization of low-boom supersonic jet. Fifteen design variables related to the shape of the fuselage and wing are encoded as genes in each chromosome. For the alternative to the expensive high-fidelity CFD analysis, Kriging approximation model is built by initial 140 CFD calculations of randomly generated points in design space. These tools are directly connected with the several GAs in this work. The parallel GA is used for single discipline case and micro-GA based GA is utilized for multi-discipline case. Both GAs show desirable speedups compared to the original GAs. Application of the methods in this work to bigger problems will demonstrate obviously improved performance. If I include other major disciplines such as structural stability, control, mission or propulsion, more realistic optimization can be performed. Therefore, the suggested efficient GAs in this paper will be well suited to find global optimum in this enlarged multi-objective design space.

Some researchers have suggested the use of a distributed GA in which Pareto dominance is applied only to neighbors within a certain region [18]. Such kind of approach can handle the problems of big computation time of ranking and possible lack of diversity previously mentioned simultaneously. The approach is efficient because Pareto dominance is applied in parallel to small groups of individuals. Diversity does not require an extra mechanism, since it naturally emerges from the distributed population. So parallelization of current non-dominated sorting GAs for multi-objective optimization will be good future work to improve the efficiency of GAs.

As stated early, a slower convergence rate of GAs at a later stage can prevent the efficiency as well. A combination of local optimizer which has more information such as gradients with the original GAs can improve the convergence history.

Those are left for the future works as well.

7. REFERENCES

1. D. E. Goldberg. "Genetic Algorithms in Search, Optimization, and Mahine Learning", Addison-Wesley, Massachusetts, 1989

2.Carlos A. Coello Coello, "A Comprehensive Survey of Evolutionary-Based Multiobjective Optimization Techniques", *Knowledge and Information Systems, An International Jouranl*, 1(3):269-308, August 1999

3. Carlos A. Coello Coello, Gregorio T. Pulido, " A Micro-Genetic Algorithm for Multiobjective Optimization", Lania-RI-2000-06, Laboratorio Nacional de Informatica Avanzada, 2000

4. J. J. Alonso, J. R. R. A. Martins, J. J. Reuther, R. Haimes, and C. A. Crawford "High-Fidelity Aero-Structural Design Using a Parametric CAD-Based Model", *16th AIAA Computational Fluid Dynamics Conference*, AIAA 2003-3429, Orlando, FL., 2003.

5. S. Choi and J.J.Alonso, "Numerical and Mesh Resolution Requirements for Accurate Sonic Boom Prediction of Complete Aircraft Configurations", accepted in *41st AIAA Aerospace Science Meeting*, AIAA 2004, Reno, Nevada

6. V. S. Gordon and D. Whitley. "Serial and parallel genetic algorithms as function optimizer", *Proceedings of the Fifth International Conference on Genetic Algorithm*s, Illinois, July 1993

7. H. Chung, S. Choi and J. J. Alonso, "Supersonic Business Jet Design using a Knowledge-Based Genetic Algorithm with an Adaptvie, Unstructured Grid Methodology", *21st Applied Aerodynamics Conference*, AIAA 2003-3791, Orlando, Florida

8. J. Sacks, W. J. Welch, T. J. Michell, and H. P. Wynn, "Design and Analysis of Computer Experiments", *Statistical Science*, Vol 4. No.4, pp. 409-453, 1989

9. Timothy W. Simpson, Timothy M. Mauery, John J. Korte and Farrokh Mistree, "Comparison of Response Surface and Kriging Models in the Multidisciplinary Design of an Aerospike Nozzle", *7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, St. Louis, Missouri, AIAA 98-4755, September 1998

10. D. Levine, "Users Guide to the PGAPack Parallel Genetic Algorithm Library", Argone National Laboratoy

11. J. Sacks, W. J. Welch, T. J. Michell, and H. P. Wynn, “Design and Analysis of Computer Experiments," *Statistical Science*, Vol 4. No.4, pp. 409-453, 1989

12. Timothy W. Simpson, Timothy M. Mauery, John J. Korte and Farrokh Mistree, “Comparison of Response Surface and Kriging Models in the Multidisciplinary Design of an Aerospike Nozzle", *7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, St. Louis, Missouri, AIAA 98-4755, September 1998

13. D. Whitely, “Cellular genetic algorithms”, *Proceedings of the 5th international conference on Genetic Algorithsm*, San Mateo, 1993

14. C. A. Coello Coello and G. T. Pulido, “A Micro-Genetic Algorithms for Multiobjective Optimization", *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO'2001)* , 2001.

15 D. L. Carroll, “Fortran Genetic Algorithm Driver,”, *http://cuaerospace.com/carroll/ga.html*, 2001

16. N.Srinivas and K. Deb, “Multiobjective optimization Using Nondominated Sorting in Genetic Algorith”, *Evolutionary Computation*, 2(3):221-248,1995

17. J. D. Knowles and D.W. Corne, “Approximating the Nondominated Front Using the Pareto Archived Evolution Strategy”, *Evolutionary Computation*, 8(2):149-172, 2000

18. J. Rowe, K. Vinsen and N. marvin, “ Parallel GAs for Multi-objective Functions”, In jarmo T. Alander, editor, *Proceedings of the Second Nordic Workshop on Genetic Algorithms and Their Applications (2NWGA)*, pages 61-70, Vaasa, Finland, August 1996