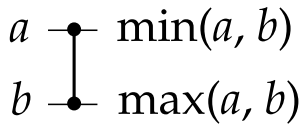
Evolving Minimal-Size Sorting Networks

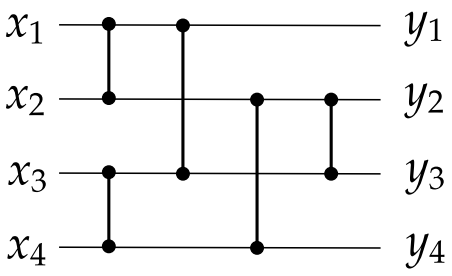
Vinod Valsalam Risto Miikkulainen

Department of Computer Sciences
The University of Texas at Austin, USA
{vkv,risto}@cs.utexas.edu

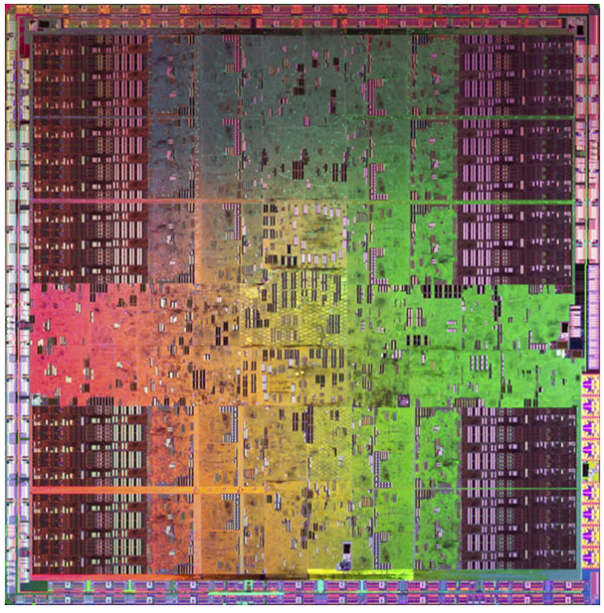
Human Competitive Results
Genetic and Evolutionary Computation Conference 2011

Sorting Networks





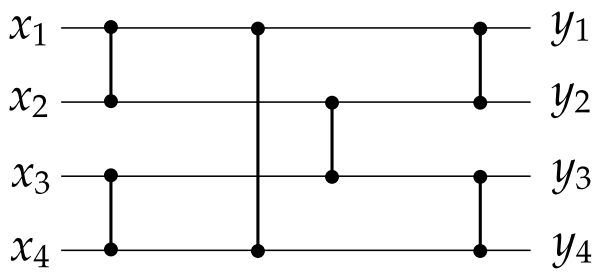
$y_1 \leq y_2 \leq y_3 \leq y_4$



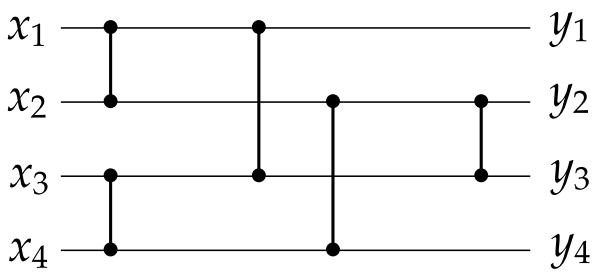
Nvidia GT200 GPU die

- Sequence of comparators for sorting n inputs
  - Data-independent sorting algorithm
- Used in parallel hardware
  - Fast sorting is crucial
  - Multi-core GPUs, switching, multi-access memories, ...

A Challenging Optimization Problem



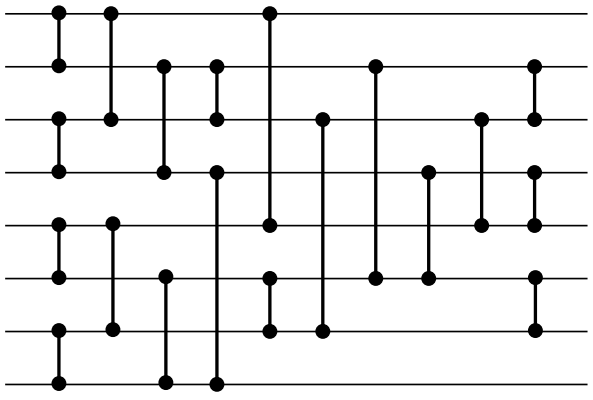
6 comparators



5 comparators

- Minimize size (i.e. number of comparators)
- Provably minimal size networks known only for $n \leq 8$
- Suboptimal heuristic methods for $n > 8$

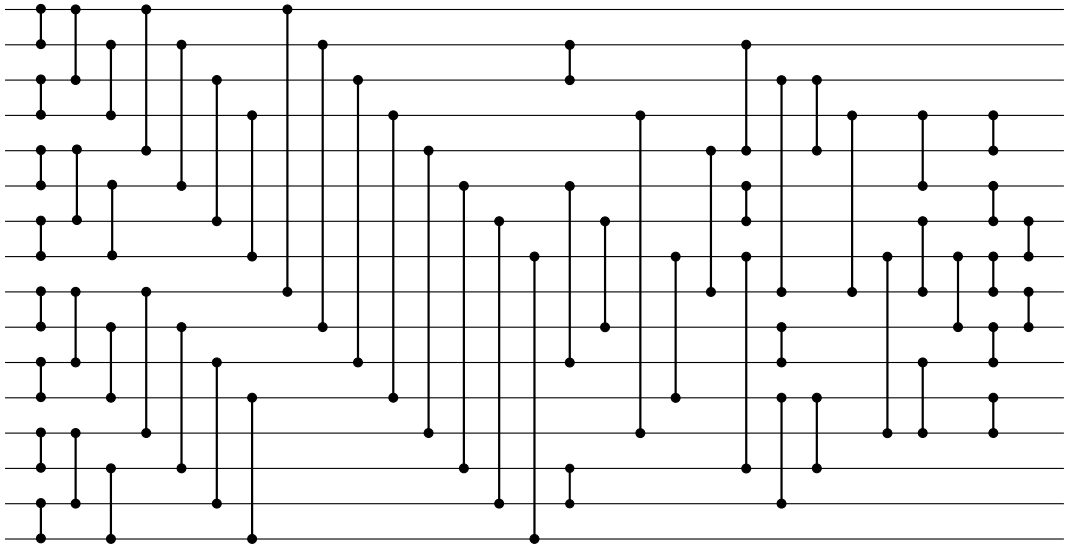
Human Designs for $n \leq 8$



Batcher's network for 8 inputs

- Provably optimal
- O'Conner and Nelson [1962], U.S. Patent 3029413
  - Hand-designed networks for $4 \leq n \leq 8$
  - 7-input network required two extra comparators
- Batcher's [1968] recursive merge algorithm
  - Optimal only for $n \leq 8$

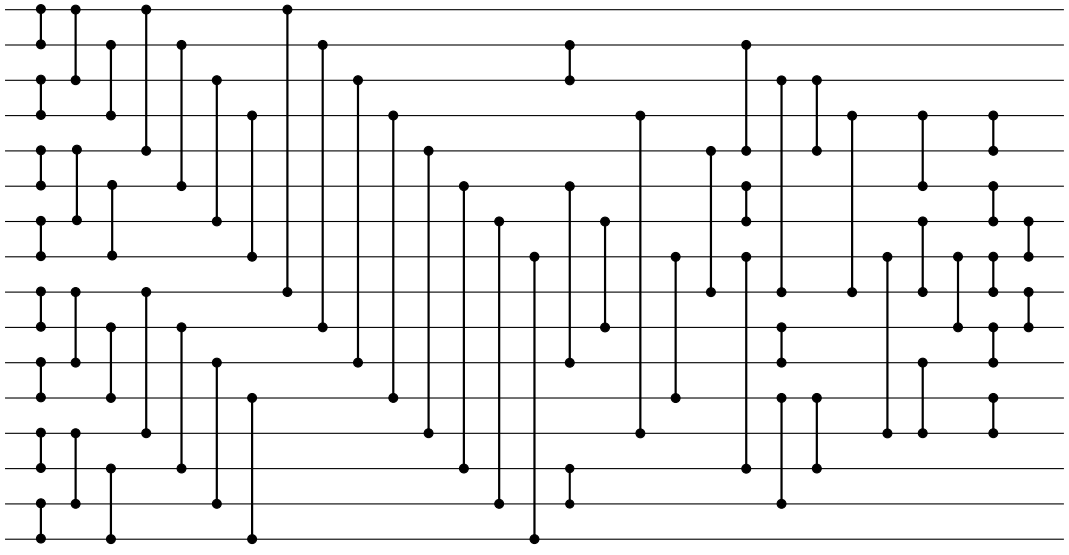
Human Designs for $8 < n \leq 16$



Hand-designed 16-input network with 60 comparators [Green, 1969]

- Optimality not known
- Constructed using special techniques
  - Human designs (except for $n = 13$ [Juille, 1995])

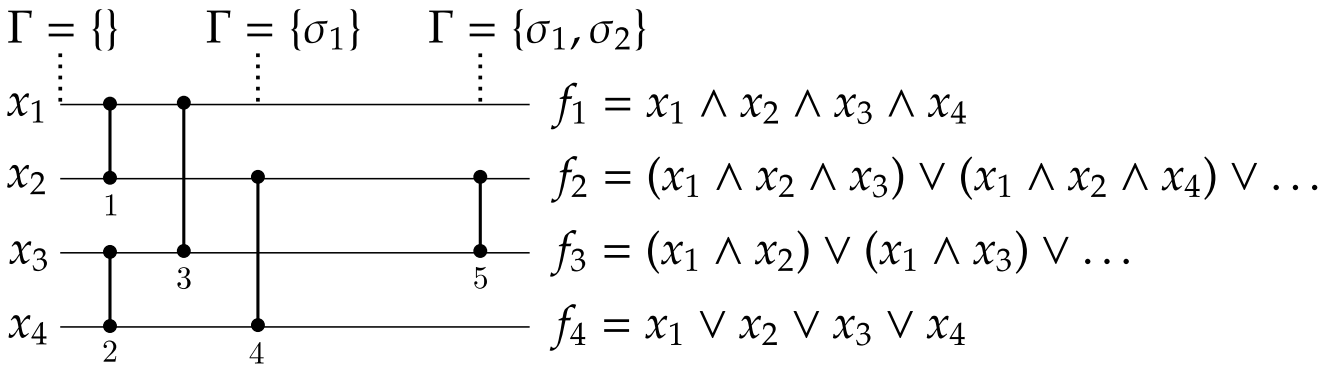
Human Designs for $n > 16$



Hand-designed 16-input network with 60 comparators [Green, 1969]

- Only merges of smaller networks are known

SENSO Approach Utilizing Symmetry and Evolution



- SENSO = Sorting ENSO method
- Add comparators greedily to build symmetry step-by-step
  - Focuses evolution on promising solutions
- Utilize an EDA (evolution) to improve greedy solutions
  - Evolution learns to anticipate minimal solutions

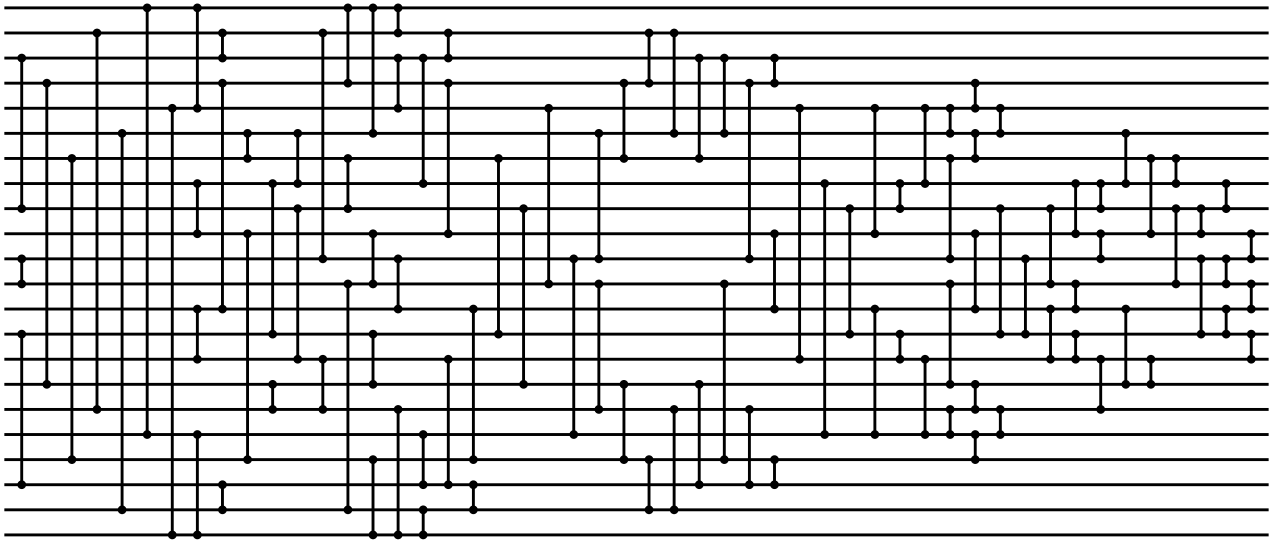
Results Evolved by SENSO

n	1	2	3	4	5	6	7	8	9	10	11	12
Prev best	0	1	3	5	9	12	16	19	25	29	35	39
SENSO	0	1	3	5	9	12	16	19	25	29	35	39

n	13	14	15	16	17	18	19	20	21	22	23	24
Prev best	45	51	56	60	73	80	88	93	103	110	118	123
SENSO	45	51	57	60	71	78	86	92	103	108	118	125

- Matched previous best results for $n < 24$, $n \neq 15$
- Improved previous best results for $n = 17, 18, 19, 20, 22$
- Potential for more: is still running!

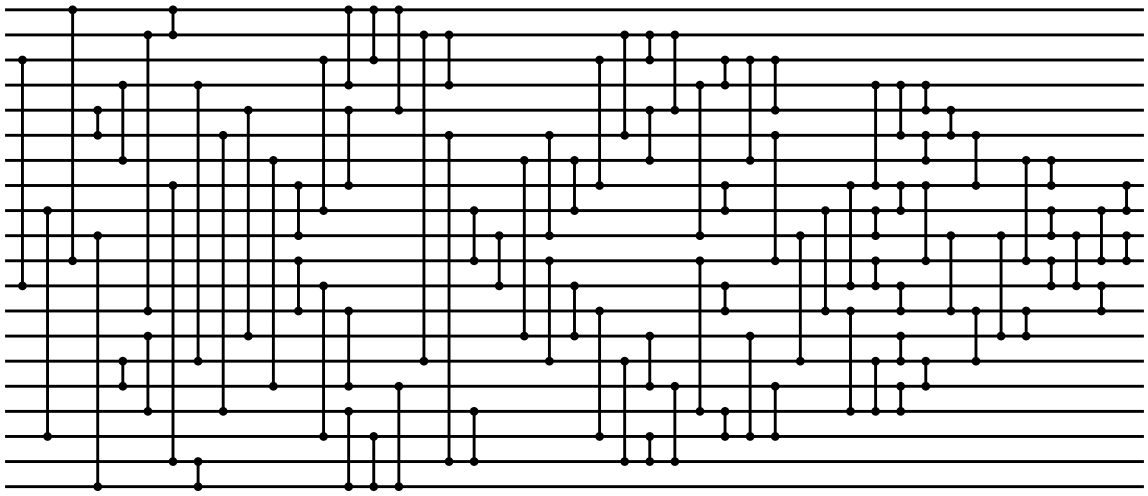
Human-Competitiveness Criterion A: Patentability



A new minimal 22-input network evolved by SENSO

- Do the results match or improve upon patented inventions?
- U.S. Patent 3029413 for the simpler $4 \leq n \leq 8$ cases
- SENSO results therefore qualify as patentable inventions

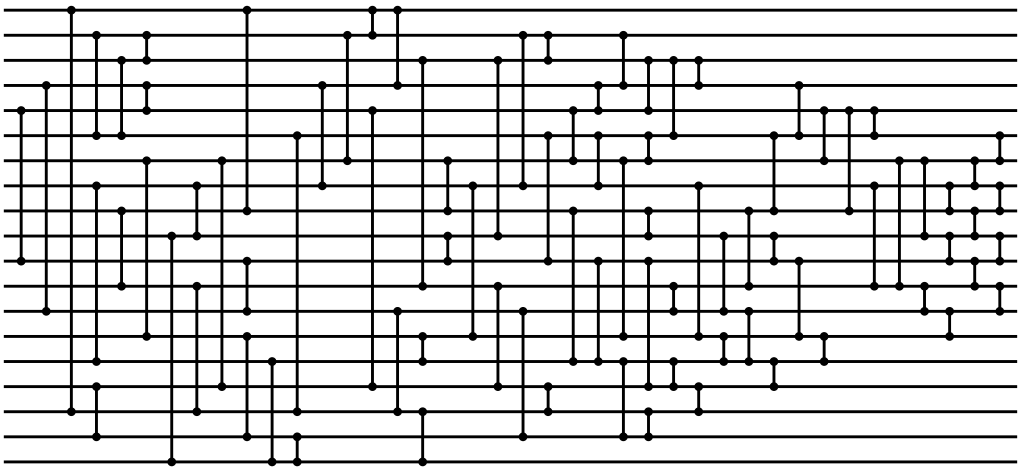
Human-Competitiveness Criteria B, D: Publishability



A new minimal 20-input network evolved by SENSO

- Are the results equal to/better than published results? (B)
- Are the results publishable as new scientific results? (D)
- Many journal publications on minimal sorting networks
  - See Knuth [1998] and Koza et al. [1999] for surveys
- SENSO improved several upper bounds $\Rightarrow$ publishable

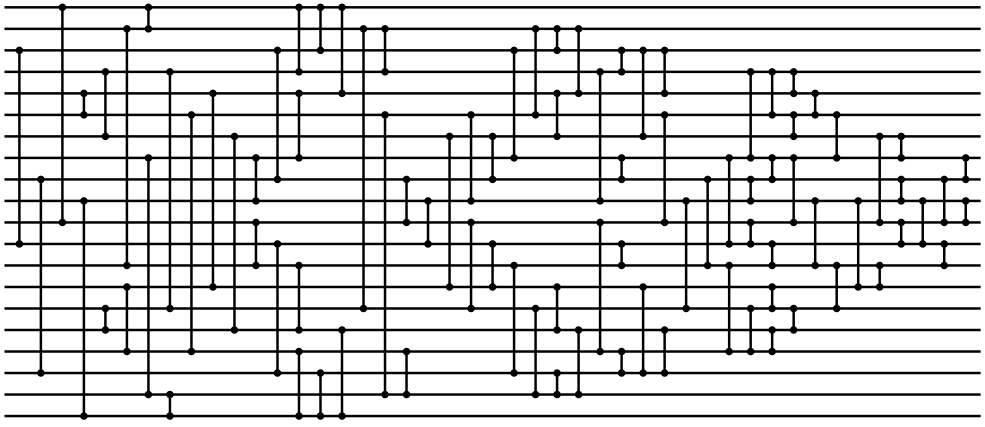
Human-Competitiveness Criteria E, F, G: Difficulty



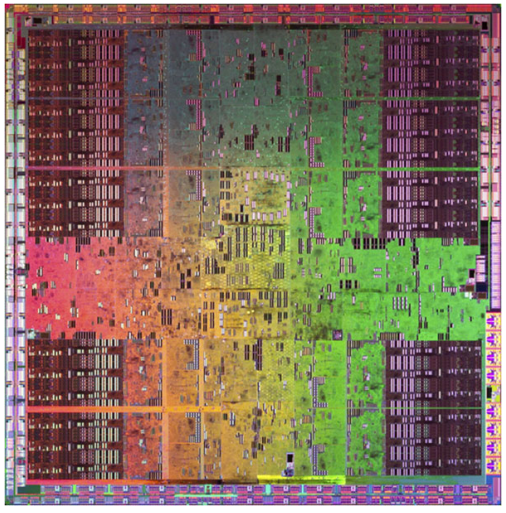
A new minimal 19-input network evolved by SENSO

- Equal to/better than a succession of human designs? (E)
- Equal to/better than an achievement in the field? (F)
- Solves a problem of indisputable difficulty? (G)
- Knuth [1998] and Koza et al. [1999] discuss the history
- SENSO scaled to larger networks and improved results

Why is this the Best Entry?



A new minimal 20-input network evolved by SENSO



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1. Not only satisfies humies criteria A, B, D, E, F, and G
2. But also improves upon half-century of theoretical results
   - Published in patents, books, peer-reviewed literature
3. And has significant practical value
   - More efficient sorting, switching, memories . . .

`http://nn.cs.utexas.edu/?sorting-code`